Self-steepening of ultrashort pulses in silicon photonic nanowires

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We study the frequency dependence of the optical nonlinearity of Si photonic nanowires (Si-PNWs) and its influence on the propagation of ultrashort optical pulses in such nanodevices. Specifically, we show that Si-PNWs present a remarkably large characteristic time associated with self-steepening effects and optical shock formation, namely, more than an order of magnitude larger than in the case of photonic crystal fibers.

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In recent years, research in the optical properties of deep subwavelength guiding structures, such as Si optical waveguides fabricated on an silicon-on-insulator material system, the so-called Si photonic nanowires (Si-PNWs), has grown at an increasing rate [1,2]. Since one of the main applications envisioned for Si-PNWs is that of chip-to-chip or intrachip ultrafast optical interconnects, significant efforts have been devoted to the understanding of the properties of picosecond [3–5] and subpicosecond [6,7] pulse dynamics in these guiding structures (for a review, see [8]).

The tight optical-field confinement achieved in Si-PNWs leads to a strong dependence of the mode profile on both the wire geometry and the material parameters and thus to a large frequency dispersion of their linear optical properties [4,9]. It is thus expected that the nonlinear properties of Si-PNWs will also show large frequency dispersion, which would play a significant role in the dynamics of ultrashort or ultrabroad optical pulses [10]. A strong frequency dependence of the optical nonlinearity leads to significant pulse reshaping, through pulse self-steepening or optical shock formation, and thus dramatically affects both the temporal and the spectral pulse evolution. In this Letter, we investigate these nonlinear optical phenomena in Si-PNWs.

An Si-PNW consists of a Si strip with width w and height h, placed on top of a SiO_2 substrate. The optical properties of such devices are primarily determined by the waveguide modes and the propagation constant, β. In addition, the dependence $\beta=\beta(\omega)$ defines the mode index, $n_{\text{eff}}=\beta c/\omega$, the group index $n_g=c/v_g$, and $\beta_2=d^2\beta/d\omega^2$, where $\beta_n=d^n\beta/d\omega^n$ is the nth order dispersion coefficient. We have determined the frequency dependence of these quantities by using commercial software, RSoft's FEMSIM, for two waveguides with $w\times h=360\text{ nm}\times 220\text{ nm}$ and $w\times h=600\text{ nm}\times 300\text{ nm}$. Our simulations show that these waveguides support only the TE-like mode E_{11}^x. Thus, the waveguide with $w\times h=360\text{ nm}\times 220\text{ nm}$ ($w\times h=600\text{ nm}\times 300\text{ nm}$) has a zero-dispersion point at $\lambda_z=1550\text{ nm}$ (1325 and 2409 nm) and anomalous group-velocity dispersion (GVD) for $\lambda<\lambda_z$ ($\lambda_{z1}<\lambda<\lambda_{z2}$).

The dynamics of an ultrashort (subpicosecond) optical pulse propagating in an Si-PNW is governed by a perturbed nonlinear Schrödinger equation [4,8]

$$i\frac{\partial u}{\partial z}+\sum_{n\geqslant 2}\frac{i^n\beta_n}{n!}\frac{\partial^n u}{\partial t^n}=-\frac{ic\kappa}{2nv_g}\alpha_{\text{FC}}u-\frac{\omega\kappa}{nv_g}\delta n_{\text{FC}}u-\frac{3\omega P\Gamma}{4\epsilon_0 A_0 v_g^2}\left(1+i\tau_s\frac{\partial}{\partial t}\right)|u|^2u, \tag{1}$$

where z (t) is the distance (time), u is the pulse envelope, P is the peak power, $A_0=wh$ is the area, α_{FC} (δn_{FC}) are the free-carrier (FC) losses (FC-induced index change) and are given by $\delta n_{\text{FC}}=-e^2N/2\epsilon_0 n\omega^2(1/m_{ce}^*+1/m_{ch}^*)$ and $\alpha_{\text{FC}}=e^3N/\epsilon_0 cn\omega^2(1/\mu_e m_{ce}^{*2}+1/\mu_h m_{ch}^{*2})$ [11], where N is the FC density, $m_{ce}^*=0.26m_0$ ($m_{ch}^*=0.39m_0$) is the effective mass of the electrons (holes), where m_0 is the mass of the electron and μ_e (μ_h) is the electron (hole) mobility. Also, Γ is defined as $\Gamma=A_0\int e^*\cdot\hat{\chi}^{(3)}\vdots ee^*e\,dA/\mathcal{J}^2$, where $\hat{\chi}^{(3)}$ is the third-order susceptibility of Si, $\mathcal{J}=\int n^2(\mathbf{r}_t)|e|^2dA$, and $e(\omega;\mathbf{r})$ are the waveguide modes. The last term in Eq. (1) describes the self-steepening of the pulse and the formation of an optical shock [12,13], an effect characterized by the shock time scale $\tau_s=\partial\ln\gamma/\partial\omega\equiv\tau_0+\tau_{wm}$, where $\gamma=3\omega P\Gamma/4\epsilon_0 A_0 v_g^2$, $\tau_0=1/\omega$ is related to the noninstantaneous response of the nonlinearity in a bulk crystal, and $\tau_{wm}(\omega)=\partial\ln(\Gamma/v_g^2)/\partial\omega$ gives the waveguide contribution, including that due to $\chi^{(3)}$. Finally, the FC dynamics are determined by [4]

$$\frac{\partial N}{\partial t}=-\frac{N}{t_c}+\frac{3P^2\Gamma''}{4\epsilon_0\hbar A_0^2v_g^2}|u|^4. \tag{2}$$

In this paper the prime and the double-prime symbols mean the real and the imaginary parts, respectively. In large waveguides the lifetime t_c is a few tens of nanoseconds; however, in Si-PNWs, owing to the fast diffusion of FCs to the edges of the waveguide, $t_c \sim 0.5$ ns [14].

The shock time coefficient is fully determined by the frequency dependence of the waveguide nonlinearity parameter γ. Thus, to characterize this nonlinear effect, we have first determined γ by using the waveguide modes and the propagation constant β and then calculated numerically the derivative with respect to ω. The coefficient γ depends on $\hat{\chi}^{(3)}$ so that the frequency dispersion of the material optical nonlinearity is incorporated in this procedure by using the experimentally measured frequency-dependent $\hat{\chi}^{(3)}$ [15]. Thus we account for the contributions of both the material and the waveguide dispersion to $\gamma(\omega)$. Note that for Si, $\hat{\chi}^{(3)}$ has two independent components, $\hat{\chi}^{(3)}_{1111}$ and $\hat{\chi}^{(3)}_{1122}$; however, a recent experiment has shown that $\hat{\chi}^{(3)}_{1111}=2.36\hat{\chi}^{(3)}_{1122}$ [16] within a broad frequency range. The remaining component is determined from the experimentally measured [15] values of the Kerr coefficient, n_2, and the two-photon absorption (TPA) coefficient, β_{TPA}, by using $n_2 = 3\hat{\chi}'^{(3)}_{\mathrm{eff}}/4\epsilon_0 c n^2$ and $\beta_{\mathrm{TPA}} = 3\omega\hat{\chi}''^{(3)}_{\mathrm{eff}}/2\epsilon_0 c^2 n^2$. For Si-PNWs fabricated along the $[1\bar{1}0]$ direction $\hat{\chi}^{(3)}_{\mathrm{eff}} = (\hat{\chi}^{(3)}_{1111}+3\hat{\chi}^{(3)}_{1122})/2$.

Figure 1(a) shows the dependence $\gamma(\lambda)$, determined both for the case in which only the waveguide dispersion is considered and for the more general case when both the material and the waveguide dispersion are included. Figure 1(a) shows that γ depends strongly on λ, with both its real and the imaginary parts decreasing more than three times within a spectral domain of 600 nm. This decrease of γ at large λ is due to the fact that as λ approaches the cutoff wavelength the mode becomes less confined in the Si core, and thus a smaller amount of power is guided within the region with optical nonlinearity. We thus expect that such large frequency dispersion of γ leads to a large "optical shock" time.

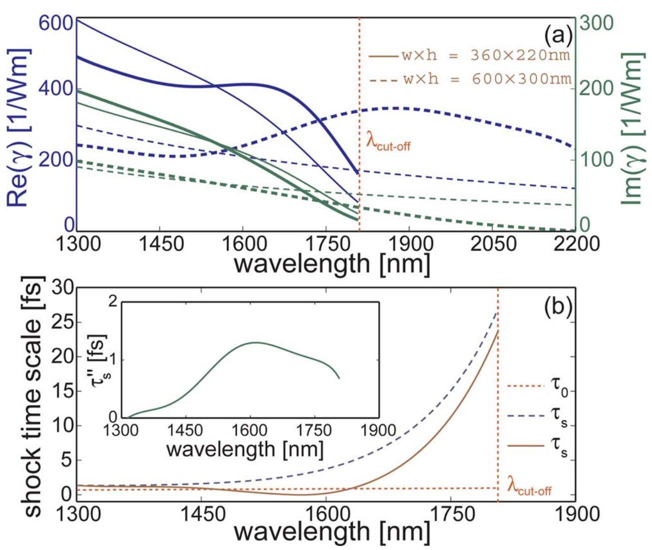

Fig. 1. (Color online) (a) Real and imaginary parts of γ versus λ, determined in the case when only the waveguide dispersion is considered (thin curves) and when both the material and the waveguide dispersion are included (thick curves). (b) Real part of τ_s when the frequency dependence of $\hat{\chi}^{(3)}$ is neglected (dashed curves) and when it is included (solid curve). The inset shows the imaginary part of τ_s versus λ.

These conclusions are confirmed by the frequency dependence of τ_s, calculated for the waveguide with $w \times h = 360$ nm $\times$ 220 nm; the results are summarized in Fig. 1(b). The most notable conclusion illustrated in Fig. 1(b) is that for Si-PNW the shock time can be as large as 25 fs, i.e., more than an order of magnitude larger than that in photonic crystal fibers (PCFs) [17]. As just discussed, it is especially large in the vicinity of the cutoff wavelength. In addition, unlike the case of optical fibers or PCFs, τ_s has a significant imaginary part, τ_s'', which stems from the frequency dispersion of $\chi''^{(3)}$ of Si. As we will show, this results in a shift of the pulse spectra toward longer wavelengths.

To study the influence of self-steepening effects on the propagation of ultrashort optical pulses in Si-PNW, we consider the evolution of a sech-shaped pulse with $\lambda_0 = 1500$ nm, $T_0 = 100$ fs, and peak power $P = 9$ W in the Si-PNW with $w \times h = 360$ nm $\times$ 220 nm. Then, the dispersion length $L_D = T_0^2/|\beta_2| = 2.5$ mm, $\gamma' = 446.5\ \mathrm{W}^{-1}\,\mathrm{m}^{-1}$, and thus the nonlinear length is $L_{\mathrm{NL}} = 1/\gamma' P = 0.25$ mm and the soliton number is $N_s = \sqrt{L_D/L_{\mathrm{NL}}} = 3.18$. To begin with, we neglect the TPA; however, the FC effects are included in our analysis. The results obtained by integrating the systems (1) and (2) are presented in Fig. 2. Thus, as shown in Figs. 2(a) and 2(b), when the waveguide contribution to τ_s is neglected, $\tau_{wm} = 0$, the pulse splits into three solitons that subsequently emit radiation at a frequency shifted by $\delta\omega = 3|\beta_2|/\beta_3$ from the soliton frequency [18], viz., $\lambda_{\mathrm{rad}} \sim 1740$ nm. Moreover, in the temporal domain, the soliton with the largest peak power is accelerated and its temporal position is shifted toward the front of the pulse, whereas in the

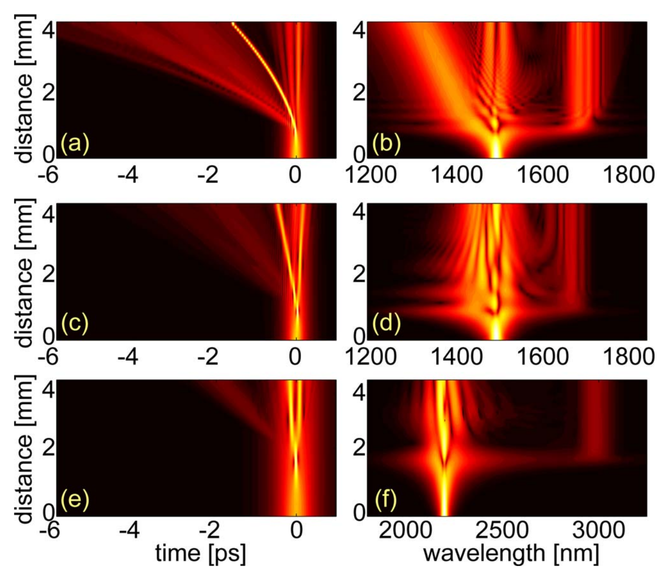

Fig. 2. (Color online) Pulse propagation in a $z = 4$ mm long Si-PNW. (a) and (b) $\tau_s = \tau_0 = 0.8$ fs ($\tau_{wm} = 0$), (c) and (d) $\tau_0 = 0.8$ fs and $\tau_{wm} = 7.5 + i0.77$ fs, (e) and (f) $\tau_s = \tau_0 = 1.17$ fs.

spectral domain, this same soliton is shifted toward the blue side of the spectrum. These features of the pulse evolution are explained by the nonlinear losses induced by FCs. Thus, Eq. (1) shows that these optical losses are proportional to $\int_{-\infty}^{t}|u(z,t')|^4 dt'$, which means that the optical loss at the front of the pulse is smaller than the loss in its tail. As the soliton propagates in the anomalous GVD region, the redshifted frequency components move more slowly than the blueshifted ones, and thus the redshifted components are absorbed more strongly. Hence the soliton is slowly shifted toward the blue side of the spectrum. In contrast, the solitons with a smaller peak power induce a much smaller nonlinear loss, and thus this shifting is less pronounced. Now consider the very different evolution of the input pulse when we include the contribution of the waveguide to τ_s. As shown in Figs. 2(c) and 2(d), the shift of the pulse now, both in time and frequency, almost vanishes. The decrease in the soliton frequency shift can be attributed to τ_s''. Thus, if $\tau_{wm}'' \neq 0$, Eq. (1) contains a term proportional to $\tau_{wm}'' u \partial |u|^2/\partial t$, a term which in optical fibers describes the intrapulse Raman scattering. As it is known, it leads to a shift of the soliton spectrum toward longer wavelengths, and thus it cancels the blueshift induced by the FCs. Note that such dynamics are unique to Si-PNW, as for optical fibers $\tau_{wm}'' = 0$, and no FCs are generated.

It is expected that the TPA would affect the pulse dynamics that we just described; however, TPA losses are negligible if $\lambda \gtrsim 2200$ nm, and thus their effects can be reduced by a proper choice of the pulse wavelength. Thus, we show in Figs. 2(e) and 2(f) the propagation of a pulse with $\lambda_0 = 2200$ nm and power $P = 9$ W in the Si-PNW with $w \times h = 600$ nm $\times$ 300 nm. To keep N_s unchanged, the pulse width is modified to $T_0 = 180$ fs. Figures 2(e) and 2(f) show that the soliton shift almost vanishes, which is because the reduced TPA leads to the generation of fewer FCs.

These predictions are also corroborated by the pulse spectrograms shown in Fig. 3, which are defined as $S(\omega,\tau) = |\int_{-\infty}^{\infty} u(z,t) u_{\mathrm{ref}}(t-\tau)\exp(i\omega t)dt|$, where u_{ref} is a reference pulse; in our calculations we used the input pulse for u_{ref}. Thus, it can be seen that the pulse spectrograms for $\tau_s = 0$ and $\tau_s = \tau_0$ are very similar, which shows that the influence of the dispersion of the bulk nonlinearity on the pulse dynamics is rather small. However, these pulse dynamics are strongly modified when we add the contribution of the waveguide. Since in this case, too, three solitons are generated, this spectrogram is topologically similar to the other two; however, as we discussed, the additional term proportional to τ_{wm}'' significantly reduces the temporal soliton shift.

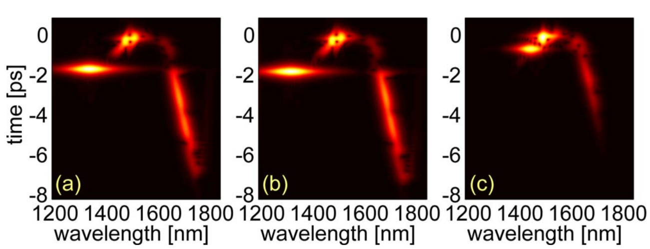

Fig. 3. (Color online) Pulse spectrograms calculated for (a) $\tau_s = 0$, (b) $\tau_0 = 0.8$ fs, $\tau_{wm} = 0$, and (c) $\tau_0 = 0.8$ fs, $\tau_{wm} = 7.5 + i0.77$ fs. In all cases the propagation distance is $z = 4$ mm.

In conclusion, we have characterized the frequency dispersion of the optical nonlinearity of Si-PNWs and its effects on ultrashort optical pulses propagating in such nanowires. We have shown that the shock time in Si-PNWs can be as large as a few tens of femtoseconds. Remarkably, since the Raman effects for ultrashort pulses propagating in Si-PNWs are negligible, this Letter shows that, unlike the case of optical fibers or PCFs, the self-steepening is the dominant higher-order nonlinear effect.

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